

Simplest Neutrino Mixing from S_4 Symmetry

R. Krishnan,^a P. F. Harrison^a and W. G. Scott^b

^a*University of Warwick,
Coventry, CV4 7AL, UK*

^b*Rutherford Appleton Laboratory,
Chilton, Didcot, Oxon, OX11 0QX, UK*

E-mail: k.rama@warwick.ac.uk, p.f.harrison@warwick.ac.uk,
w.g.scott@rl.ac.uk

ABSTRACT: In 2004, two of us proposed a texture, the “Simplest” neutrino mass matrix, which predicted $\sin \theta_{13} = \sqrt{2\Delta m_{sol}^2/3\Delta m_{atm}^2}$ and $\delta_{CP} = 90^\circ$. Using today’s measured values for neutrino mass-squared differences, this prediction gives $\sin^2 2\theta_{13} \simeq 0.086_{-0.006}^{+0.003}$, compared with a measured value of $\sin^2 2\theta_{13} = 0.093 \pm 0.010$. Here we present a specific model based on S_4 symmetry leading to this successful texture in the context of the type-1 see-saw mechanism, assuming Majorana neutrinos. Similar predictions are obtained relating θ_{13} to the light neutrino masses, which are in accord with current experimental limits and testable at future experiments. Large CP asymmetries remain a generic prediction of the texture.

Contents

1	Introduction	1
2	The group S_4 and the μ-τ rotated basis	3
3	The model	5
4	Fitting the model with experimental data	7
5	Summary	9
6	Appendix A: Flavon Potentials	9

1 Introduction

Leptonic mixing is characterised by two large mixing angles, $\theta_{12} \simeq 35^\circ$ and $\theta_{23} \simeq 45^\circ$, and one small angle, θ_{13} . For several years, the data on neutrino oscillations were compatible with $\theta_{13} = 0$, and the data together were approximated by the tribimaximal (TBM) mixing matrix, proposed in 2002 [1]. TBM has been used by many authors as a starting point for model building. The Daya Bay Reactor Neutrino Experiment [2] has recently measured the value for the reactor mixing angle, $\sin^2 2\theta_{13} = 0.092 \pm 0.016$ (stat.) ± 0.005 (syst.), showing definitively that θ_{13} is non-zero. Similarly, the RENO Experiment [3] also made a compatible measurement, $\sin^2 2\theta_{13} = 0.113 \pm 0.013$ (stat.) ± 0.019 (syst.). In models of leptonic mixing based on discrete symmetries starting with TBM, it is possible to generate non-zero θ_{13} by introducing higher order corrections, but in the generic case, the deviations produced should be of the same order for all three mixing angles [4]. However, since the experimentally allowed deviation of θ_{12} from its TBM value, $\sin^2 \theta_{12} = 1/3$, is small, it is difficult to generate the relatively large experimental value of θ_{13} in this way.

Anticipating an eventual non-zero value for θ_{13} , two of us proposed several generalisations of the TBM texture in 2002 [5], in which the condition $\theta_{13} = 0$ was relaxed in various ways. For example, in a basis in which the charged lepton mass matrix is diagonal, a hermitian neutrino mass matrix which has μ - τ reflection symmetry [6] and democracy (each of its rows and columns sums to a common value [7]) leads to “tri χ maximal” mixing (T χ M) [5]. The most general hermitian mass matrix giving T χ M takes the form:

$$M_H = a \begin{pmatrix} 1 & ik & -ik \\ -ik & 0 & 1+ik \\ ik & 1-ik & 0 \end{pmatrix} + b \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} + c \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1.1)$$

where k , a , b , and c are real parameters.

The mass matrix arising from the Majorana mass term for the neutrinos should be complex-symmetric. Therefore we wish to determine the general complex-symmetric mass matrix that generates T χ M. One way to achieve this is to multiply the matrix M_H by the μ - τ exchange operator P to get the complex-symmetric matrix M_S :

$$M_H P = M_S = a \begin{pmatrix} 1 & -ik & ik \\ -ik & 1+ik & 0 \\ ik & 0 & 1-ik \end{pmatrix} + b \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + c \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad (1.2)$$

where $P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.

The only free mixing angle in T χ M is the angle θ_{13} which is uniquely determined by the real parameter k in the matrix M_S . The four mixing observables in this texture are given by:

$$|U_{e2}|^2 = \frac{1}{3}, \quad \sin^2 \theta_{23} = \frac{1}{2}, \quad \sin^2 \theta_{13} = \frac{1}{3} \left(1 - \frac{1}{\sqrt{1+3k^2}} \right); \quad \delta_{CP} = \frac{\pi}{2}. \quad (1.3)$$

We remark that with $\delta_{CP} = 90^\circ$, T χ M corresponds to maximal CP violation, for given values of the three real mixing angles. The T χ M mixing matrix, U , defined by Eq. (1.3), which diagonalises the matrix M_S of Eq. (1.2), also diagonalises each of its three terms independently. The first term of M_S alone is in fact, sufficient to generate T χ M, having three distinct eigenvalues. The second and the third terms of M_S give two degenerate and three degenerate eigenvalues respectively. As with any complex-symmetric matrix, M_S can be diagonalised using a unitary matrix and its transpose to give real positive eigenvalues. In other words:

$$U^\dagger M_S U^* = \text{Diag}(|a\sqrt{1+3k^2} - b + c|, |a + 2b + c|, |-a\sqrt{1+3k^2} - b + c|). \quad (1.4)$$

A special case of the T χ M texture, known as ‘‘Simplest’’ neutrino mixing was proposed in 2004 [8] (after having been introduced and discussed briefly already in 2002 [5]) by setting $b = 0$ in the texture of Eq. (1.1). Its eigenvalues are given by the RHS of Eq. (1.4) with $b = 0$. Simplest neutrino mixing yields an exact and very straightforward relation between the reactor mixing angle (see Eq. (1.3)) and the eigenvalues, e_i :

$$\sin^2 \theta_{13} = \frac{2(e_2 - e_1)}{3(e_3 - e_1)}. \quad (1.5)$$

In the original publications [5, 8], this texture was proposed for $M_\nu^2 := M_\nu M_\nu^\dagger$, in which case the eigenvalues are the neutrino masses-squared, resulting in the very successful prediction:

$$\sin \theta_{13} = \sqrt{\frac{2}{3} \frac{\Delta m_{\text{sol}}^2}{\Delta m_{\text{atm}}^2}}. \quad (1.6)$$

$$\text{ie. } \sin^2 2\theta_{13} = 0.086_{-0.006}^{+0.003} \quad (\text{Predicted in 2002/2004 [5, 8]}) \quad (1.7)$$

$$\text{cf. } \sin^2 2\theta_{13} = 0.093 \pm 0.010 \quad (\text{Measured in 2012 [2, 3]}). \quad (1.8)$$

In view of the very encouraging phenomenological success of the “Simplest” mixing hypothesis, we here propose a model for it based on the symmetric group of degree four (S_4). In the model, however, we make the following two changes with respect to the original hypothesis: the “Simplest” mass matrix form is adopted for the mass matrix itself (as opposed to its hermitian square); in order to exploit the type-I see-saw mechanism a Majorana mass term is assumed (coupling between two heavy right-handed neutrinos). Thus, we get a Majorana neutrino mass matrix of the following “Simplest” complex-symmetric form:

$$M_\nu(\text{Majorana}) = a \begin{pmatrix} 1 & -ik & ik \\ -ik & 1+ik & 0 \\ ik & 0 & 1-ik \end{pmatrix} + c \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (1.9)$$

In the following section we construct such a Majorana mass matrix assuming symmetry under the S_4 group. The neutrino Dirac mass matrix (coupling between the left-handed and the right-handed neutrinos) is assumed to be proportional to the identity. We show that this model has a phenomenology compatible with experiment, and we use it to predict the masses of the light neutrinos.

2 The group S_4 and the μ - τ rotated basis

S_4 , the group of permutations of four objects, is the symmetry group of the cube and the octahedron. In an abstract form, the group has the presentation

$$\langle \mathbf{a}, \mathbf{b} | \mathbf{a}^2 = \mathbf{b}^3 = (\mathbf{ab})^4 = e \rangle, \quad (2.1)$$

where \mathbf{a} , \mathbf{b} and \mathbf{ab} represent the orientation-preserving rotations of a cube through angles π , $\frac{2\pi}{3}$ and $\frac{\pi}{2}$ respectively. This is shown in Fig. 1, where axis_{*a*} and axis_{*b*} correspond to the generators \mathbf{a} and \mathbf{b} respectively. The character table for the S_4 group can be found in [9].

We may work in a basis with:

$$\mathbf{a} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad (2.2)$$

as is used frequently in model building. In this basis the coordinate system is oriented such that the coordinate axes are normal to the faces of the cube, as shown in Fig. 1(a). So the x , y and z coordinate axes are the symmetry axes of $\frac{\pi}{2}$ -rotations. In a model constructed with a neutrino triplet $(\nu_e, \nu_\mu, \nu_\tau)$ defined parallel to the coordinate axes (x, y, z) in the above basis, ν_e , ν_μ and ν_τ are simply the invariant eigenstates (eigenstates with eigenvalue equal to +1) of these particular $\frac{\pi}{2}$ -rotations¹. This choice of eigenstates is straightforward, and is the one used in most models using this group so far. It is however, by no means the only choice, and there is no reason why we should not define the flavour basis states in a different way. To construct a model for deriving the “Simplest” texture, it will prove

¹Abstractly they are also the eigenvectors of the corresponding elements of the three-element conjugacy class C_3 (π -rotations about axes passing through face centres).

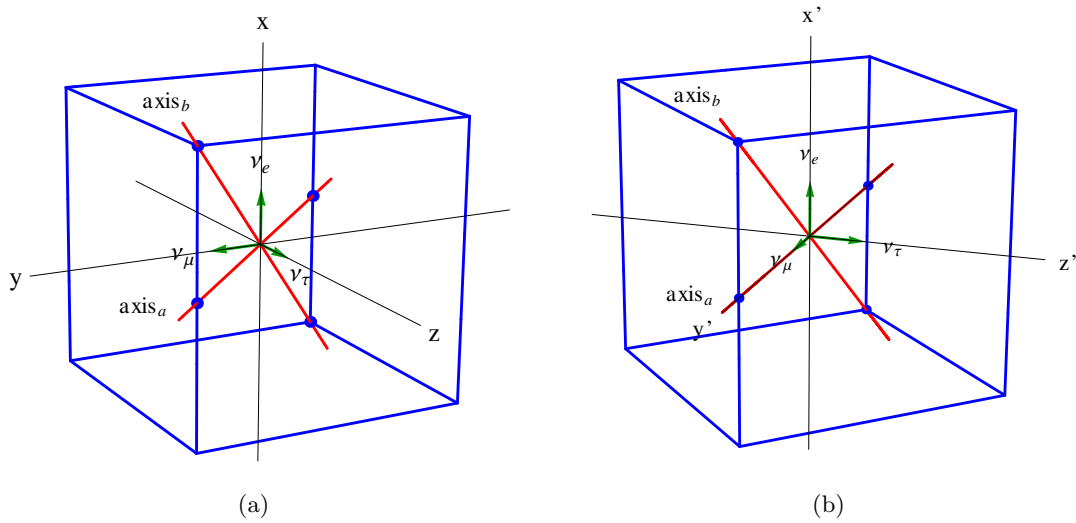


Figure 1. Octahedral symmetry in the μ - τ rotated basis (x', y', z') .

useful to define the ν_μ and ν_τ basis states rotated by an angle $\frac{\pi}{4}$ relative to the x , y and z coordinate axes defined above, using the rotation matrix,

$$R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (2.3)$$

The matrix R represents a rotation about the x axis by an angle $\pi/4$, relative to the cube. We also rotate the y and z coordinate axes to align with the new ν_μ and ν_τ flavour basis states respectively. In the rotated coordinate system, (x', y', z') in Fig. 1(b), we have the group generators:

$$\mathbf{a} \rightarrow R \cdot \mathbf{a} \cdot R^\dagger = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad \mathbf{b} \rightarrow R \cdot \mathbf{b} \cdot R^\dagger = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}. \quad (2.4)$$

The state ν_e is unchanged and still corresponds to the $\frac{\pi}{2}$ -rotation symmetry of the cube about the x -axis. However, ν_μ and ν_τ are no longer invariant eigenstates of $\frac{\pi}{2}$ -rotations, but rather are invariant under π rotations of the cube, as may be seen in Fig. 1(b). We call this new basis, the μ - τ rotated basis.

The three-dimensional representation of S_4 corresponding to the rotational symmetries of the cube is denoted by $\mathbf{3}'$ here. Thus the neutrino triplet $(\nu_e, \nu_\mu, \nu_\tau)$ belongs to the $\mathbf{3}'$ representation. A Majorana mass term contains two neutrino fields, and thus it is of interest to consider the tensor product decomposition of two $\mathbf{3}'$ s. This decomposition is as follows:

$$\mathbf{3}' \times \mathbf{3}' = \mathbf{1} + \mathbf{2} + \mathbf{3} + \mathbf{3}', \quad (2.5)$$

where $\mathbf{1}$ is the trivial representation, and in the μ - τ rotated basis we have:

$$\begin{aligned}\chi_1 &= \frac{1}{\sqrt{3}}(\nu_e \cdot \nu_e + \nu_\mu \cdot \nu_\mu + \nu_\tau \cdot \nu_\tau), & \chi_2 &= \begin{pmatrix} -\sqrt{\frac{2}{3}}\nu_e \cdot \nu_e + \frac{1}{\sqrt{6}}\nu_\mu \cdot \nu_\mu + \frac{1}{\sqrt{6}}\nu_\tau \cdot \nu_\tau \\ \frac{1}{\sqrt{2}}(\nu_\mu \cdot \nu_\tau + \nu_\tau \cdot \nu_\mu) \end{pmatrix} \\ \chi_3 &= \begin{pmatrix} \frac{1}{\sqrt{2}}(\nu_\mu \cdot \nu_\mu - \nu_\tau \cdot \nu_\tau) \\ \frac{1}{\sqrt{2}}(\nu_\tau \cdot \nu_e + \nu_e \cdot \nu_\tau) \\ \frac{1}{\sqrt{2}}(\nu_e \cdot \nu_\mu + \nu_\mu \cdot \nu_e) \end{pmatrix}, & \chi'_3 &= \begin{pmatrix} \frac{1}{\sqrt{2}}(\nu_\mu \cdot \nu_\tau - \nu_\tau \cdot \nu_\mu) \\ \frac{1}{\sqrt{2}}(\nu_\tau \cdot \nu_e - \nu_e \cdot \nu_\tau) \\ \frac{1}{\sqrt{2}}(\nu_e \cdot \nu_\mu - \nu_\mu \cdot \nu_e) \end{pmatrix} = 0,\end{aligned}\tag{2.6}$$

where the bi-linears $\chi_1, \chi_2, \chi_3, \chi'_3$ transform as $\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{3}'$ respectively. The product $\nu_i \cdot \nu_j$ is the Lorentz invariant product of the right-handed neutrino Weyl spinors. Obviously the terms in χ'_3 in Eq. (2.6) vanish.

We now assume three types of flavons, ϕ_1, ϕ_2 and ϕ_3 which transform as $\mathbf{1}, \mathbf{2}$ and $\mathbf{3}$ respectively, allowing us to write an invariant mass term:

$$\text{Inv} = c_1 \chi_1 \phi_1 + c_2 \chi_2^T \phi_2 + c_3 \chi_3^T \phi_3,\tag{2.7}$$

where c_1, c_2 and c_3 are constants. Once the flavons acquire specific forms of vacuum expectation values (vevs), the required mass matrix can be obtained from the invariant mass term given in Eq. (2.7). Suppose the flavons get vevs

$$\langle \phi_1 \rangle = 1, \langle \phi_2 \rangle = \begin{pmatrix} -\frac{1}{2}, \frac{\sqrt{3}}{2} \end{pmatrix}, \langle \phi_3 \rangle = (1, 1, -1),\tag{2.8}$$

the mass matrix obtained will be

$$M = c_1 I + \frac{c_2 \sqrt{3}}{2\sqrt{2}} \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 1 \\ 0 & 1 & -\frac{1}{3} \end{pmatrix} + \frac{c_3}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix},\tag{2.9}$$

which is in the same form as Eq. (1.9), assuming c_1 and c_2 to be real and c_3 to be imaginary (where here the trace is removed from the second term and is absorbed into the first). The details of how to obtain the above mentioned flavon vevs using the minimisation of flavon potentials are given in Appendix A.

3 The model

The model is constructed in a Standard Model framework with the addition of heavy right-handed neutrinos. Through a type-1 seesaw mechanism, light Majorana neutrinos are produced. The fermion and flavon content of the model with representations to which they belong is given in the Table 1. We construct a diagonal mass matrix for the charged leptons using the mass term given by Eq. (3.1). The C_2 flavour symmetries $C_{2e}, C_{2\mu}$ and $C_{2\tau}$ ensure that the flavons $\phi'_{3e}, \phi'_{3\mu}$ and $\phi'_{3\tau}$ couple to only e_R, μ_R and τ_R respectively. In order to construct the Dirac mass term containing the right-handed neutrinos and the left-handed

lepton doublets, we postulate a singlet flavon ϕ_1^- . We also introduce another C_2 symmetry C_{2D} to allow only the singlet flavon ϕ_1^- to enter the Dirac mass term, Eq. (3.2), and thus to make the neutrino Dirac mass matrix proportional to the identity. The Majorana mass term containing the right-handed neutrinos, Eq. (3.3), leads to a mass matrix of the form given by Eq. (2.9) as explained in the previous section. The Standard Model Higgs field is assigned to the trivial representation of S_4 .

	e_R	μ_R	τ_R	L	ν_R	ϕ_1^-	ϕ_1	ϕ_2	ϕ_3	ϕ'_{3e}	$\phi'_{3\mu}$	$\phi'_{3\tau}$
S_4	1	1	1	3'	3'	1	1	2	3	3'	3'	3'
C_{2e}	-1	1	1	1	1	1	1	1	1	-1	1	1
$C_{2\mu}$	1	-1	1	1	1	1	1	1	1	1	-1	1
$C_{2\tau}$	1	1	-1	1	1	1	1	1	1	1	1	-1
C_{2D}	-1	-1	-1	-1	1	-1	1	1	1	1	1	1

Table 1. The flavour structure of the model. L are the three left-handed lepton weak isospin doublets and ν_R are the three right-handed heavy neutrinos. The **3'** representations are in the μ - τ rotated basis and the **2** and the **3** representations are in the basis given by the tensor product expansion in Eqs. (2.5, 2.6).

For the charged leptons, the mass term is of the form

$$\left(y_e L^\dagger e_R \phi'_{3e} + y_\mu L^\dagger \mu_R \phi'_{3\mu} + y_\tau L^\dagger \tau_R \phi'_{3\tau}\right) \frac{H}{\Lambda} + H.C. \quad (3.1)$$

where H is the standard model Higgs, Λ is the cut-off scale and the y_i are coupling constants. After the spontaneous breaking of the weak gauge symmetry and the flavour symmetry with the flavons, ϕ'_{3e} , $\phi'_{3\mu}$ and $\phi'_{3\tau}$, getting vevs of $(1, 0, 0)^T$, $(0, 1, 0)^T$ and $(0, 0, 1)^T$ respectively, we get the required masses m_e , m_μ and m_τ for the charged leptons. The Dirac mass term for the neutrinos takes the form

$$y_w L^\dagger \nu_R \frac{\phi_1^-}{\Lambda} \tilde{H} + H.C. \quad (3.2)$$

where \tilde{H} is the conjugate Higgs and y_w is a coupling. We also have the Majorana mass term for the neutrinos:

$$(y_1 \chi_1 \phi_1 + y_2 \chi_2^T \phi_2 + i y_3 \chi_3^T \phi_3) \frac{1}{\Lambda}, \quad (3.3)$$

where the χ_i are given by the expressions in Eqs. (2.6) and the y_i ($i = 1, 2, 3$) are couplings leading to very heavy right-handed Majorana masses. The flavons ϕ_1 , ϕ_2 and ϕ_3 getting the vevs given in Eqs. (2.8) and also having $\langle \phi_1^- \rangle = 1$ result in the following 6×6 mass matrix M for the neutrinos:

$$\nu_\alpha^T M \nu^\alpha \quad (3.4)$$

$$M = \begin{pmatrix} 0 & M_{\text{Dir}} \\ M_{\text{Dir}} & M_{\text{Maj}} \end{pmatrix}, \quad \nu = \begin{pmatrix} \nu_L^* \\ \nu_R \end{pmatrix} \quad (3.5)$$

with $\nu_L = (\nu_e, \nu_\mu, \nu_\tau)$ the left-handed neutrino flavour eigenstates where $M_{\text{Dir}} = \frac{y_w}{2\Lambda} I$ and M_{Maj} is of the required form given in Eqs. (1.9) and (2.9). Here, M_{Maj} is at a very high mass scale and M_{Dir} is of order the weak scale, so that the seesaw mechanism comes into play. It can be shown [10] that the effective mass matrix, M_{ss} , generated via the seesaw mechanism for the left-handed light neutrinos is in the form

$$M_{\text{ss}} = -M_{\text{Dir}} M_{\text{Maj}}^{-1} M_{\text{Dir}}. \quad (3.6)$$

The matrix iU^* diagonalises M_{ss} giving light neutrino masses proportional to $\frac{1}{e_1}, \frac{1}{e_2}, \frac{1}{e_3}$ where e_1, e_2 and e_3 are the eigenvalues of M_{Maj} .

It is to be noted that we may rotate the fermion and the flavon fields from the μ - τ rotated basis to the conventional basis and obtain the same physical result, even though in the conventional basis the charged-lepton mass matrix will not be diagonal and the Majorana mass term for the neutrinos will not be in the ‘‘Simplest’’ texture. Therefore the use of μ - τ rotated basis in the model ensures that the charged-lepton mass matrix is diagonal and the Majorana mass term is in the ‘‘Simplest’’ texture.

4 Fitting the model with experimental data

The squared differences of the light neutrinos masses are known experimentally, $m_2^2 - m_1^2 = 75.9 \pm 2.1 \text{ meV}^2$, $|m_3^2 - m_2^2| = 2430 \pm 130 \text{ meV}^2$. The eigenvalues of the Majorana mass matrix, M_{Maj} from Eq. (1.9), are

$$e_1 = c + a\sqrt{1 + 3k^2}, \quad e_2 = c + a, \quad e_3 = c - a\sqrt{1 + 3k^2} \quad (4.1)$$

(a, c and k are real in our model, given our earlier assumptions). Since the light neutrino masses are inversely proportional to these eigenvalues, we get

$$\begin{aligned} \frac{m_2^2 - m_1^2}{m_3^2 - m_2^2} &= \pm \frac{75.9 \pm 2.1}{2430 \pm 130} = \frac{\frac{1}{e_2} - \frac{1}{e_1}}{\frac{1}{e_3} - \frac{1}{e_2}} \\ &= -\frac{(r - s)^2(1 - s)(1 + 2r + s)}{(r + s)^2(1 + s)(1 + 2r - s)} \end{aligned} \quad (4.2)$$

where $r = c/a$ and $s = \sqrt{1 + 3k^2}$. Using Eq. (1.3) we can calculate the parameter k given the reactor mixing angle θ_{13} . Substituting the value of k in Eq. (4.2) and solving for the parameter r , we can predict the values of the light neutrino masses. Eq. (4.2) is cubic in r giving three separate real solutions for normal hierarchy and one for inverted hierarchy. One of the normal hierarchy solutions gives the wrong sign for the solar mass-squared difference, leaving three remaining solutions. Thus the light neutrino masses predicted by the model fall into three sets. These results are shown in Fig. 2 where the best fit values are used. The error ranges of the mass of the neutrino eigenstate ν_1 for the three solutions are shown in Fig. 3. For solution 2 we do not consider a mass above 100 meV in order to keep our prediction compatible with the cosmological upper limit of the masses of the neutrinos. The WiggleZ Dark Energy Survey [11] gives the strongest cosmological

limit so far, $\sum m_\nu < 290$ meV. It should be emphasised that in tri χ maximal mixing (T χ M), given the three mixing angles θ_{12} , θ_{23} , θ_{13} , maximal CP violation ($\delta_{CP} = 90^\circ$) is always guaranteed. The positive and negative signs of the parameter k correspond to the CP -violating phase $\delta_{CP} = \pm 90^\circ$.

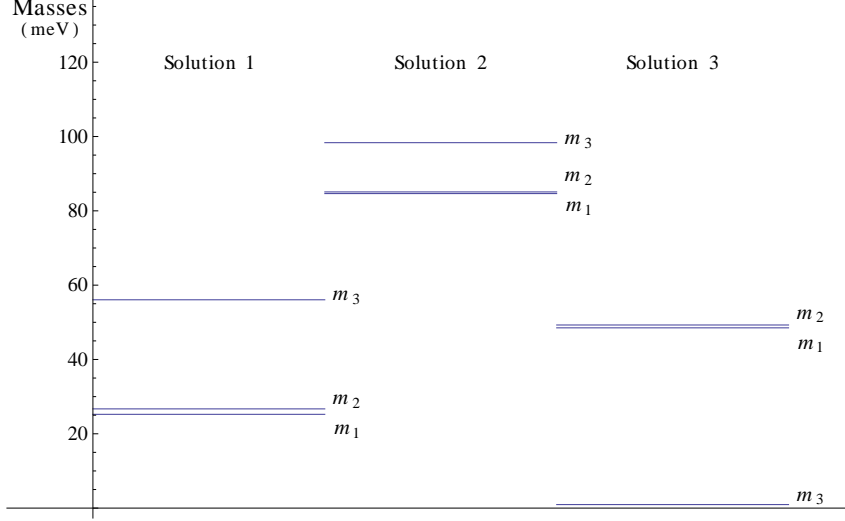


Figure 2. The predicted values of the neutrino masses corresponding to the best fit ($m_2^2 - m_1^2 = 75.9$ meV², $|m_3^2 - m_2^2| = 2430$ meV², $\sin^2 2\theta_{13} = 0.098$). Case 1 ($r = 0.4101$) and case 2 ($r = 14.452$) are in normal hierarchy. Case 3 ($r = -1.0405$) is in inverted hierarchy.

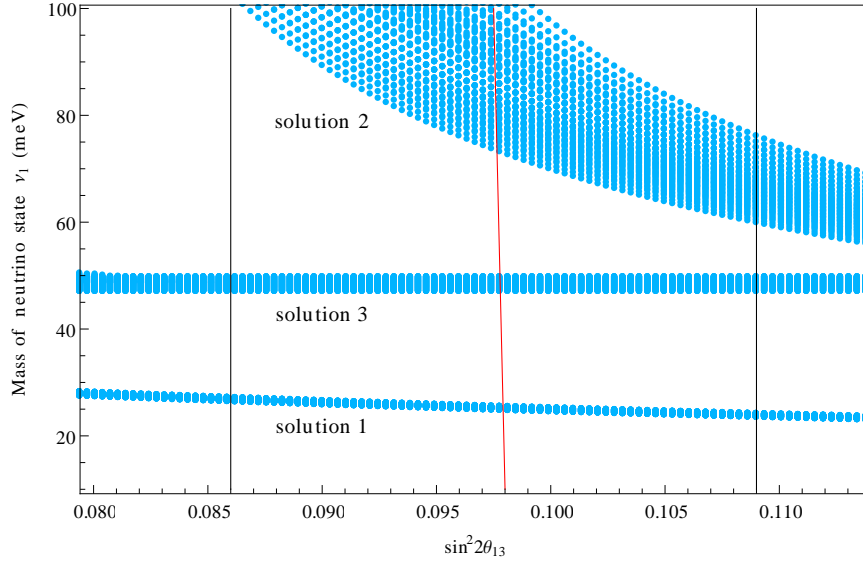


Figure 3. The predicted value of m_1 (the mass of the neutrino eigenstate ν_1) vs the measured value of $\sin^2 2\theta_{13}$. The finite thickness of the bands is due to the errors in the measurement of the neutrino mass-squared differences. The red and the black lines indicate the best fit value and the errors on $\sin^2 2\theta_{13}$ respectively.

5 Summary

We use the “Simplest” texture of neutrino mass matrix to explain the recently measured non-zero reactor mixing angle. When used as a hermitian square of the mass matrix, this texture successfully predicts the observed reactor angle (with the help of the neutrino mass-squared differences). When used as a complex-symmetric Majorana mass term, the constraint among the masses and the mixing resulting from the texture can be used to predict the unknown lightest neutrino mass (with the help of the reactor angle). We get three solutions which are compatible with the measured mass-squared differences. In the model we exploit the “ μ - τ rotated basis”, introduced here for the first time, to obtain the required texture for the mass matrix. Since the mixing is tri χ maximal, the CP -violating phase is predicted to be $\pm\frac{\pi}{2}$. Large CP violation such as this is potentially testable in future experiments.

This work was supported by the UK Science and Technology Facilities Council (STFC). Two of us (PFH and RK) acknowledge the hospitality of the Centre for Fundamental Physics (CfFP) at the Rutherford Appleton Laboratory. RK acknowledges support from CfFP and the University of Warwick.

6 Appendix A: Flavon Potentials

Defining the flavon $\phi_2 = (\phi_2^1, \phi_2^2)$, we can construct a third degree invariant, $-(\phi_2^1)^3 + 3(\phi_2^2)^2\phi_2^1$. Along with the term $(\phi_2^1)^2 + (\phi_2^2)^2$ (which is basically a U_1 invariant), we can construct the potential

$$V(\phi_2) \propto -(\phi_2^1)^3 + 3(\phi_2^2)^2\phi_2^1 + ((\phi_2^1)^2 + (\phi_2^2)^2)^3 \quad (6.1)$$

leading to a vev of $\langle\phi_2\rangle = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$, as required.

For the flavon ϕ_3 , it is easier to work in the non- μ - τ -rotated basis. Defining $\phi_3 = (\phi_3^1, \phi_3^2, \phi_3^3)$ in this basis, S_4 invariants can be easily constructed by summing up even powers of ϕ_3^1 , ϕ_3^2 and ϕ_3^3 symmetrically. A potential constructed in this way is given below:

$$V(\phi_3) \propto ((\phi_3^1)^8 + (\phi_3^2)^8 + (\phi_3^3)^8) + p((\phi_3^1)^6 + (\phi_3^2)^6 + (\phi_3^3)^6) + q((\phi_3^1)^4 + (\phi_3^2)^4 + (\phi_3^3)^4) \\ + r((\phi_3^1)^2 + (\phi_3^2)^2 + (\phi_3^3)^2) + s((\phi_3^1)^2(\phi_3^2)^2 + (\phi_3^3)^2(\phi_3^2)^2 + (\phi_3^1)^2(\phi_3^3)^2) \quad (6.2)$$

for real p , q , r and s under some constraints. This results in $\langle\phi_3\rangle = (1, \sqrt{2}, 0)$ in the non-rotated basis, corresponding to $\langle\phi_3\rangle = (1, 1, -1)$, in the μ - τ rotated basis, as required. Using the extremisation condition of zero first order derivatives applied at the point $\phi_3 = (1, \sqrt{2}, 0)$, we get the following constraints:

$$r = -60 - 21p - 6q, \quad (6.3)$$

$$s = 28 + 9p + 2q. \quad (6.4)$$

To ensure that the extrema points are minima, we need to impose the condition of positive definite Hessian matrix. This gives the following inequalities:

$$(4 + p) > 0, \tag{6.5}$$

$$(18 + 5p + q) > 0, \tag{6.6}$$

$$(208 + 72p + 9p^2 - 8q) > 0. \tag{6.7}$$

References

- [1] P. F. Harrison, D. H. Perkins, and W. G. Scott, *Tri-bimaximal mixing and the neutrino oscillation data*, *Phys. Lett. B* **530** (2002) 167–173, [[hep-ph/0202074](#)].
- [2] F. P. An *et al.* (Daya Bay Collaboration), *Observation of electron-antineutrino disappearance at daya bay*, *Phys. Rev. Lett.* **108** (2012) 171803, [[arXiv:1203.1669](#)].
- [3] J. K. Ahn *et al.* (RENO Collaboration), *Observation of reactor electron antineutrinos disappearance in the reno experiment*, *Phys. Rev. Lett.* **108** (2012) 191802, [[arXiv:1204.0626](#)].
- [4] G. Altarelli and F. Feruglio, *Discrete flavor symmetries and models of neutrino mixing*, *Rev. Mod. Phys.* **82** (2010) 2701–2729, [[arXiv:1002.0211](#)].
- [5] P. F. Harrison and W. G. Scott, *Symmetries and generalisations of tri-bimaximal neutrino mixing*, *Phys. Lett. B* **535** (2002) 163–169, [[hep-ph/0203209](#)].
- [6] P. F. Harrison and W. G. Scott, *Mu-tau reflection symmetry in lepton mixing and neutrino oscillations*, *Phys. Lett. B* **547** (2002) 219–228, [[hep-ph/0210197](#)].
- [7] P. F. Harrison and W. G. Scott, *Permutation symmetry, tri-bimaximal neutrino mixing and the s_3 group characters*, *Phys. Lett. B* **557** (2003) 76–86, [[hep-ph/0302025](#)].
- [8] P. F. Harrison and W. G. Scott, *The simplest neutrino mass matrix*, *Phys. Lett. B* **594** (2004) 324–332, [[hep-ph/0403278](#)].
- [9] H. Ishimori, T. Kobayashi, H. Ohki, Y. Shimizu, H. Okada, and M. Tanimoto, *Non-abelian discrete symmetries in particle physics*, *Prog. Theor. Phys. Suppl.* **183** (2010) 1–163, [[arXiv:1003.3552](#)].
- [10] A. Y. Smirnov, *Seesaw enhancement of lepton mixing*, *Phys. Rev. D* **48** (1993) 3264–3270, [[hep-ph/9304205](#)].
- [11] S. Riemer-Sorensen *et al.* (WiggleZ Collaboration), *The wigglez dark energy survey: Cosmological neutrino mass constraint from blue high-redshift galaxies*, *Phys. Rev. D* **85** (2012) 081101, [[arXiv:1112.4940](#)].